NAME: _____

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M427J Quiz 6

Problem 1. **[3 pts]** Write the eigenvalues of the matrix T. Hint: no work is required.

$$T = \begin{pmatrix} 1 & 3 & 1 \\ & 2 & 2 \\ & & 1 \end{pmatrix}$$

Problem 2. **[5 pts]** Suppose A is a 3×3 real matrix whose only eigenvalue is 0. Also assume that $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ is a basis for the set of solutions to $A\vec{x} = 0$, and that $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ is a basis for the set of solutions to $A^2\vec{x} = 0$. You are given that $A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$. Find the general solution to $\frac{d}{dt}\vec{x} = A\vec{x}$.

Problem 3. **[2 pts]**

(a) In one line, show that if $\vec{x}$ is a solution to $\frac{d}{dt}\vec{x} = A\vec{x}$, then so is $\vec{y} = \frac{d}{dt}\vec{x}$.

(b) Using part (a), prove that if $\vec{x} = e^{\lambda t}\vec{B}(t)$ is a solution to $\frac{d}{dt}\vec{x} = A\vec{x}$, then so is $\vec{y} = e^{\lambda t}\frac{d}{dt}\vec{B}(t)$. Hint: the product rule from calculus holds even though $\vec{B}(t)$ is a vector function. Hint 2: linear combinations of solutions are solutions.