

Analytic limits

As was suggested by a student in class, if $f, g : (a, b) \rightarrow \mathbb{R}$ are analytic at $t_0 \in (a, b)$ and $\lim_{t \rightarrow t_0} f(t)/g(t)$ exists, then f/g (with the limiting value filled in at t_0) is analytic at t_0 .

We can use this to more simply analyze the example from class. We were trying to show $\frac{te^t}{e^t-1}$ is analytic. By L'Hôpital's rule,

$$\lim_{t \rightarrow 0} \frac{te^t}{e^t - 1} = \lim_{t \rightarrow 0} \frac{e^t + te^t}{e^t} = \lim_{t \rightarrow 0} (1 + t) = 1$$

so the function can be filled in with the value 1 at $t = 0$ in order to make it analytic. I apologize for making this more complicated than necessary in discussion.